Master thesis for Spring-Summer 2025

Connecting discrete and continuous neural network models

Keywords: Neural networks, differential equations, neural ODE, formal relationships

Context and description

Université Gustave Eiffel is a multi-campus national university in France which was born in 2020 to cover all societal challenges generated by the design of the cities of tomorrow. Part of the COSYS ("Components and Systems") department of this university, the ESTAS laboratory "Evaluation and Safety of Automated Transport Systems" develops methods, techniques and tools intended to facilitate and improve the analysis and assessment of the safety functions of guided transport systems. A recent interest of ESTAS lab is on the safety verification of artificial intelligence functions in autonomous vehicles.

Although neural networks are currently the most popular models in most branches of the AI community, a new AI model called neural ODE (neural ordinary differential equation) has recently been introduced and has shown promising results [1]. A neural ODE can be seen as a continuous generalization of a discrete neural network into a differential equation, or conversely a neural network can be seen as a Euler discretization of a neural ODE model.
The goal of this Master thesis is to establish formal relationships (such as simulation relations in [2]) between the behavior of a discrete neural network and the corresponding continuous neural ODE. Such relationships could then be used to guarantee the safe behavior of one of the models after proceeding to the safety verification of the other model.

The main objectives of this Master thesis are the following:
- Short literature review of existing neural ODE models and their relation to discrete neural networks, and on formal behavioral relationships between models.
- Establishing formal relations between the discrete and continuous neural models.
- Combining these formal relationships with existing verification algorithms (for neural networks or neural ODE) to deduce the safety of one model based on the safety verification of the other.

Desired profile

- The applicant should be in their last year of Master or engineering school, in applied mathematics (preferred), control engineering, computer science, artificial intelligence, or other related fields.
- Good knowledge on modelling and analysis of differential equations, both linear and non-linear.
- Knowledge on artificial intelligence and neural networks is a plus, but the internship will mainly focus on the theoretical analysis of the mathematical models, and not on their design and training.
- Good level in English (supervision will be done in English)

Details

- Location: ESTAS lab, on the Lille campus of Université Gustave Eiffel, at Villeneuve d'Ascq
- Duration: 6 months, with flexible dates in Spring-Summer 2025
- Compensation: 3,90€/hour (around 570€/month on average)
- Application procedure: send your CV, motivation letter and copy of recent grades to

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References

[1] Chen, R. T., Rubanova, Y., Bettencourt, J., & Duvenaud, D. K. (2018). Neural ordinary differential equations. *Advances in neural information processing systems*, *31*.
[2] Tabuada, P. (2009). *Verification and control of hybrid systems: a symbolic approach*. Springer Science & Business Media.